An Analysis of Proportional, PI, and Sliding Mode Controls in Cruise Control, Anti Lock Brake, and Automated Highway Systems
ME 131 Final Term Project
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Abstract
Over the course of this paper, a longitudinal control system’s upper and lower level designs are analyzed utilizing cruise control, anti-lock brake, and adaptive cruise control (automated highway) systems. A version of sliding mode control is utilized in each of these designs, amongst the commonly used P and PI control systems, each compared for robustness and a number of other factors. Utilizing ODE45 and Simulink in Matlab, each system is modified to get the most desired results, and it will be determined which system is best for each application.
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1 Introduction

1.1 Cruise Control

Standard cruise control systems (sometimes known as speed control or autocruise) consist of a throttle control based on driver input. The specified throttle position is held until the driver disengages the cruise control either by turning it off manually or by applying the brakes. The modern cruise control was invented in 1945 by Ralph Teetor and was first implemented on the 1958 Chrysler Imperial. The system takes its speed input either from the rotating driveshaft, speedometer cable, wheel speed sensor from the engine's RPM, or from internal speed pulses. Cruise control systems calculate the ground speed by using the driveshaft rotations and use a solenoid to vary the throttle position when needed in order to maintain the vehicle's speed [4].

The cruise control model is hierarchical in structure with an upper and lower level controller. The upper controller determines the desired acceleration, while the lower level controller is responsible for tracking the desired acceleration by controlling throttle input. The real-time throttle input is tracked using vehicle dynamic models, engine maps, and nonlinear control synthesis techniques. When designing the upper level controller it is important to specify that the steady state error must be zero. In doing so, the vehicle will converge to the desired speed input set by the driver. This will ensure robustness and integrity of the system. It is also important to specify a small time constant in order to minimize time delay between user input and action of the system. To optimize this process we will be looking at the difference between a P and PI controller for the upper layer controller. It is also possible to have an upper layer controller which uses a PID controller, but that system was not explored in this term paper. The lower layer controller will be modeled with a sliding mode controller as part of the vehicle dynamics [7].

A somewhat simplified cruise control system will be analyzed, operating under two main assumptions: that zero slip exists between the tires and the road, and that the torque converter is locked (operated normally by a clutch). These are reasonable assumptions to make and thus this model, although simplified, can still be used for design of cruise control systems without losing too much integrity.

1.2 Antilock Brake Systems

Losing control of the wheels while braking can be terrifying for a driver and potentially dangerous. Wheels that have locked due to over-braking will begin to skid, removing control from the driver. When one's vehicle's tires start to skid, the normal response would be to apply more brake pressure, making the problem worse. The correct action would be to let up on the brakes to remove some brake pressure, and ultimately
increase the braking force on the wheels. To prevent accidents due to the wheels locking and skidding as a result of over-braking, many companies have come up with solutions. The first Anti-lock Brake System (ABS) controller was designed by Chrysler and implemented on the 1971 Imperial. That same year, General Motors introduced a rear-wheel ABS system on their Cadillac models that had rear-wheel drive, while Nissans President became Japan’s first car with electronic ABS. In the late 1980s, ABS was added to motorcycles, and now electronic ABS systems come standard with the majority of cars on the road today [1].

The typical ABS system has wheel sensors on each wheel, an electronic control unit (ECU), and hydraulic valve actuators within the hydraulic brake lines. If the ECU detects one wheel rotating significantly slower than the other wheels (a sign of imminent lock), the valves are actuated to reduce braking force, thus decreasing tire slip. The same is true if one wheel is rotating faster than the other three wheels, the ABS system can increase brake pressure by opening the hydraulic valve.

An Anti-lock Brake System controls the vehicle by preventing the wheels from locking up during periods of hard braking, as well as by maximizing the braking forces applied by each tire. This is done by maximizing the longitudinal slip ratio when possible, and preventing the tire forces from surpassing this value. Physically, the ABS system limits the amount of brake torque applied to the wheels, in order to prevent over-braking, during which the wheels lock up and the braking force decreases. By comparing wheel speed to vehicle speed (both of which can be measured), the locking of the tires can be detected and the ABS control system can be turned on, which will take control of the brakes and decrease the braking force in order to provide more tire force.

The ABS control law is derived using sliding mode control. This set of simulations is used to employ ABS control on a vehicle that will see varying friction coefficients, to aid in determining the robustness of the control system. The main objectives of this report are to create a simulation model based on dynamic and control equations, to examine the reliability and robustness of the simulation to varying friction coefficients experienced by the tires of the vehicle (simulating different surfaces and changing road conditions).

1.3 Adaptive Cruise Control

One way of increasing throughput and improving safety on highways is to implement some kind of adaptive cruise control in the form of an automated highway system. Vehicles can be grouped into semi-automated platoons which would accelerate and decelerate simultaneously (and would be string stable), significantly increasing the capacity of roads. The driver reaction time would be taken out of the equation, thus essentially
eliminating the response time. It can clearly be seen how this could revolutionize highway travel.

This paper is going to explore adaptive cruise control (ACC) in the way of constant spacing, for a platoon of vehicles. Adaptive cruise control can be extremely beneficial when used in conjunction with an automated highway system (AHS). This effectively removes the driver position, leaving the cars fully automated. This ACC system has the potential to greatly increase safety on the highways, as it completely removes driver error from the situation.

We will be comparing proportional versus proportional-integral control for the lead vehicle, while the rest of the platoon will be modeled with sliding mode controllers. In order to determine proper constants, the proportional and PI controls will be tested on the lead vehicle to see how it behaves when accelerating to its top desired speed. Based off of ride quality (by looking at velocity overshoot and the time it takes to reach this top speed), constants will be chosen and implemented into the lead vehicle of a platoon. A velocity matrix is chosen to determine what speeds the lead vehicle must reach over time and simulations will be done to compare the PI and the proportional platoons. A similar test is done with a platoon that is twice as large as the original to see if any patterns develop. Then, the spacing maintained by the sliding control will be changed to see if any overshoot can be diminished. With the newly optimized control system, the proportional and PI platoons will be run through a real world simulation. The stability of the system will be verified over the course of these tests.

2 Dynamic Model

In order to more properly focus on the controller designs of each system, a "new" dynamic model was not derived. Instead, Rajamani's derivation of a vehicle is utilized below, and described in detail. For more information, please see reference [7].

By utilizing longitudinal dynamics, a car can be simulated given a sequence of input engine torque. The dynamic model for longitudinal dynamics can be derived from a car experiencing multiple forces on a slope.
The variables are defined below:

\( F_{xf} \)  Front tire longitudinal force
\( F_{xr} \)  Rear tire longitudinal force
\( F_{aero} \)  Aerodynamic drag force
\( R_{xf} \)  Force of rolling resistance on front tires
\( R_{xr} \)  Force of rolling resistance on rear tires
\( m \)  Vehicle mass
\( g \)  Acceleration due to gravity (9.81m/s)
\( \theta \)  Angle of inclination of the road

Using a rudimentary force balance, the following primary equation can be determined:

\[
m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg \sin(\theta) \quad (1)
\]

### 2.1 Longitudinal Tire Force

\( F_{xf} \) and \( F_{xr} \), the friction forces that act on the front and rear tires, depend on three things: slip ratio, normal load, and road friction coefficient. As defined in Rajamani ([7]), slip ratio is defined as:

\[
\sigma_x = \frac{r_{eff} w_w - \dot{x}}{x} \quad \text{during braking.} \quad (2)
\]

\[
\sigma_x = \frac{r_{eff} w_w - \dot{x}}{r_{eff} w_w} \quad \text{during acceleration.} \quad (3)
\]
Utilizing these equations, combined with a longitudinal tire stiffness parameter $C_\sigma$ (which may differ from the front and rear tires), the tire force can be calculated:

$$F_x = C_\sigma \sigma_x \quad (4)$$

### 2.2 Aerodynamic Force

Aerodynamic force is represented as:

$$F_{\text{aero}} = \frac{1}{2} \rho C_d A_F (\dot{x} + V_{\text{wind}})^2 \quad (5)$$

$\rho$ is the mass density of air, $C_d$ is the drag coefficient, $A_F$ is the area of the front of the vehicle ($A_F = 1.6 + 0.00056(m - 765))$, $V_x$ is the vehicle velocity, and $V_{\text{wind}}$ is the wind velocity (negative for a tail wind) [7].

### 2.3 Rolling Resistance

Roughly proportional to the normal force on each tire, rolling resistance is modeled as such:

$$R_{xf} + R_{xr} = f(F_{zf} + F_{zr}) \quad (6)$$

where $f$ is the rolling resistance coefficient. Equation (6) is an estimation, as $\Delta x$ (see Figure 2) is not easily measured. A typical value of $f = 0.015$ is normally used for a passenger car. [7]
Figure 2: Rolling Resistance Diagram [7]

Normally, the moment \( F_z(\Delta x) \) is balanced due to the rolling resistance force \( R_x r_{stat} \) (\( r_{stat} \) is the statically loaded radius of the tire).

\[
R_x = \frac{F_z(\Delta x)}{r_{stat}} \tag{7}
\]

### 2.4 Normal Tire Forces

The following figure shows all of the variables that are taken into account when determining the normal tire forces \( F_{zf} \) and \( F_{zr} \).

- \( h \): Height of center of gravity
- \( h_{aero} \): Height of where \( F_{aero} \) acts
- \( l_f \): Distance from front tire to center of gravity
- \( l_r \): Distance from rear tire to center of gravity
- \( r_{eff} \): Effective radius of tires

Taking moments about the front tire (contact point) and solving for \( F_{zf} \):

\[
F_{zf} = \frac{-F_{aero}h_{aero} - m\ddot{x}h - mgh\sin(\theta) + mgl_r\cos(\theta)}{l_f + l_r} \tag{8}
\]

Similarly, solving for \( F_{zr} \) by taking moments about the rear tire:

\[
F_{zr} = \frac{F_{aero}h_{aero} + m\ddot{x}h + mgh\sin(\theta) + mgl_f\cos(\theta)}{l_f + l_r} \tag{9}
\]
As the vehicle increases in speed, the load changes (decreases on the front tires, rear tire increases). [7]

3 Controller Design
3.1 Cruise Control
3.1.1 Upper Level Controller

First order filter model for the vehicle:

$$\tau \frac{d}{dt} \ddot{x} + \ddot{x} = \ddot{x}_{des}$$  \hspace{1cm} (10)

The Proportional Control Law has the form of:

$$\ddot{x}_{des} = -k_p(v - v_{des})$$  \hspace{1cm} (11)

By substituting (11) into (10), assuming $v_{des} = constant$ and $v(t) = e^M$:

$$\tau \frac{d^2v}{dt^2} + \frac{dv}{dt} + k_p = k_p v_{des}$$  \hspace{1cm} (12)

The plant model and P controller can be represented in the Laplace domain.

$$P(s) = \frac{v}{\ddot{x}_{des}} = \frac{1}{s(\tau s + 1)}$$  \hspace{1cm} (13)
CONTROLLER DESIGN

\[ C(s) = k_p \]  \hspace{1cm} (14)

Similarly to the PI control, the PID control can be represented as a transfer function from \( v \) to \( v_{des} \):

\[ T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{k_p}{\tau s^2 + s + k_p} \]  \hspace{1cm} (15)

PI control law of the form:

\[ \dot{x}_{des} = -k_p(v - v_{des}) - k_i \int (v - v_{des}) dt \]  \hspace{1cm} (16)

By substituting (16) into (10), assuming \( v_{des} = \text{constant} \) and \( v(t) = e^M \), we can find the ODE relating \( v_{des} \) to \( v \).

\[ \tau \frac{d^3 v}{dt^3} + \frac{d^2 v}{dt^2} + k_p \frac{dv}{dt} + k_i v = k_p \dot{v}_{des} + k_i v_{des} \]  \hspace{1cm} (17)

The plant model and PI controller can be represented in the Laplace domain.

\[ P(s) = \frac{v}{\dot{x}_{des}} = \frac{1}{s(\tau s + 1)} \]  \hspace{1cm} (18)

\[ C(s) = k_p + \frac{k_i}{s} \]  \hspace{1cm} (19)

Similar to the proportional control, the PI control can be represented as a transfer function from \( v \) to \( v_{des} \):

\[ T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{k_p s + k_i}{\tau s^3 + s^2 + k_p s + k_i} \]  \hspace{1cm} (20)

3.1.2 Lower Level Controller

Using the following system state vector, and representing the velocity of the vehicle as \( \dot{x} = r_{eff} Rw_e \) (where \( R \) is the gear ratio) with initial conditions \( x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \):

\[ \begin{bmatrix} w_e \\ m_a \end{bmatrix} \]  \hspace{1cm} (21)

Relating engine speed \( w_e \) to \( T_{net} \) (total combustion torque):

\[ \dot{w}_e = \frac{T_{net} - [c_a R_{eff}^3 w_e^2 - R(reffR_x)]}{J_e} \]  \hspace{1cm} (22)
In order to get the acceleration of the vehicle ($\ddot{x}$) equal to the desired acceleration ($\ddot{x}_{des}$), the following must be true:

$$T_{netdes} = \frac{J_e}{r_{eff}R} \ddot{x}_{des} + [c_a R^3 r_{eff}^3 w_e^2 + R(ref f R_x)]$$ (23)

Dynamics of the mass of air:

$$\dot{m}_a = MAX \cdot TC(\alpha) \cdot PRI - \dot{m}_{ao}$$ (24)

Sliding mode controller that ensures $m_{ades}$ will track $m_a$:

$$TC(\alpha) = \frac{1}{MAX \cdot PRI} (\dot{m}_{ao} + \dot{m}_{ades} - \eta(m_a - m_{ades}))$$ (25)

The following equations are also used in the simulations:

$$\dot{m}_{ao} = c_1 \eta_{vol} \cdot m_a \cdot w_e$$ (26)

$$\eta_{vol} = (24.5 w_e - 3.1 \cdot 10^4) m_a^2 + (-0.167 w_e + 222) m_a + (8.1 \cdot 10^{-4} w_e + 0.352)$$ (27)

$$T_{net} = kc_1(24.5 w_e - 3.1 \cdot 10^4) m_a^3 + kc_1(-0.167 w_e + 222) m_a^2 + kc_1(8.1 \cdot 10^{-4} w_e + 0.352) m_a - 0.1056 w_e + 15.1$$ (28)

### 3.2 Anti Lock Brakes

The error that we want to drive to zero is chosen as the sliding surface $S$.

$$S = \lambda_{des} - \lambda_{act}$$ (29)

Where $\lambda \equiv$ slip ratio $(1 - \frac{w_e}{w_v})$. By differentiating this sliding surface expression, and setting it equal to $-\eta S$,

$$\dot{S} = \dot{\lambda}_{des} - \dot{\lambda}_{des} = -\eta S$$ (30)

Where $\eta$ is a controller gain chosen to drive the error to zero. After combining expressions using dynamic equations and solving for the control input:

$$P_{br} = -\frac{[-\eta(\lambda_{des} - \lambda_{act}) + \dot{\lambda}_{des}] I_a_v - F_{f_r} R_r}{A_{piston} r_{piston}}$$ (31)
3 CONTROLLER DESIGN

3.3 Automated Highway System

The lead vehicle of this highway system will be utilizing both a proportional and PI controller to determine which is more robust, utilizing the dynamic vehicle model outlined above. The proportional controller used is a simple one, outlined by the following control law:

$$\ddot{x}_{ldes} = -k_p(\dot{x}_l - \dot{x}_{ldes})$$  \hspace{1cm} (32)

The value of $k_p$ was varied throughout the simulations, and a value of 0.15 was deemed sufficient to get the least amount of overshoot when coming up to a cruising speed of 65 miles per hour (29 m/s). After 30 seconds the lead vehicle stabilizes to its desired speed.

The PI controller is similar to the proportional controller. It is outlined by the following control law:

$$\ddot{x}_{ldes} = -k_p(\dot{x}_l - \dot{x}_{ldes}) - k_i \int (\dot{x}_l - \dot{x}_{ldes})dt$$  \hspace{1cm} (33)

Through similar simulations as (32), the values $k_p = 0.01$ and $k_i = 0.2$ were determined to be the choices that provided the best ride quality.

In order to have constant spacing between each vehicle of the platoon, a sliding mode controller must be used in conjunction with the lead controller. This sliding control will constantly check the spacing between each vehicle and adjust as necessary to ensure the sliding surface goes to zero. Rajamani recommends the following sliding surface:

$$S_i = \dot{\epsilon}_i + \frac{w_n}{\zeta + \sqrt{\zeta^2 - 1}} \frac{1}{1 - C_1} \epsilon_i + \frac{C_1}{1 - C_1} (V_i - V_l)$$  \hspace{1cm} (34)

$V_i$ and $V_l$ each refer to the velocity of the lead and $i^{th}$ vehicle. The following sliding condition will drive the sliding surface to 0:

$$\dot{S}_i = -\lambda S_i$$  \hspace{1cm} (35)

$\lambda = w_n(\zeta + \sqrt{\zeta^2 - 1})$, $\epsilon_i = x_i - x_{i-1} + L_i$, and $x_{ides} = x_{i-1} - L_i$, $L_i$ being a constant that includes the length of the following vehicle and $\epsilon$ being the spacing error. Using the preceding information, the control law can be solved for, and reads as follows:

$$\ddot{x}_{ides} = (1 - C_1)\ddot{x}_{i-1} + C_1\ddot{x}_l - (2\zeta - C_1(\zeta + \sqrt{\zeta^2 - 1}))w_n\epsilon_i - (\zeta + \sqrt{\zeta^2 - 1})w_nC_1(V_i - V_l) - w_n^2\epsilon_i$$  \hspace{1cm} (36)
4 Simulations

4.1 Cruise Control

The following constants (unless otherwise stated) are utilized in the simulations. Both a P and PI controller are used.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$498636$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$0.112$</td>
</tr>
<tr>
<td>$m$</td>
<td>$1900$</td>
</tr>
<tr>
<td>$g$</td>
<td>$9.81$</td>
</tr>
<tr>
<td>$r_{eff}$</td>
<td>$0.305$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$0.015$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$1.2$</td>
</tr>
<tr>
<td>$Cd$</td>
<td>$0.3$</td>
</tr>
<tr>
<td>$A_f$</td>
<td>$1.6 + 0.00056(m - 765)$</td>
</tr>
<tr>
<td>$R_x$</td>
<td>$\mu mg$</td>
</tr>
<tr>
<td>$R$</td>
<td>$0.867$</td>
</tr>
<tr>
<td>$c_a$</td>
<td>$0.5\rho CdA_f$</td>
</tr>
<tr>
<td>$J_e$</td>
<td>$0.1454$</td>
</tr>
<tr>
<td>$MAX$</td>
<td>$0.1843$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$10$</td>
</tr>
<tr>
<td>PRI</td>
<td>$1$</td>
</tr>
</tbody>
</table>

For simplicity, $PRI = 1$ as opposed to the normally used $PRI = 1 - e^\frac{Pm}{Patm} - 1$ and $V_{wind} = 0$.

The values optimal values of $k_p$ and $k_i$ for a PI controller were derived as shown below.

In steady state the right side of the ODE relating $v_{des}$ to $v$ for a PI system is zero. Thus we can say that:

$$\tau \frac{d^3v}{dt^3} + \frac{d^2v}{dt^2} + k_p \frac{dv}{dt} + k_i * v = 0.$$  \hfill (37)

Represented in the Laplace domain we get the characteristic equation for the PI system.

$$\tau s^3 + s^2 + k_p * s + k_i = 0$$  \hfill (38)

In order to ensure stability, $\tau, k_p, k_i > 0$ and $k_p > \tau k_i$. If we assume that the system has eigenvalues at $s = -0.6 \pm 0.3 * i$ and $s = -2$, we can solve for the required values of $k_p$ and $k_i$ to ensure stability. Based on the eigenvalues given, the required characteristic equation is:

$$s^3 + 3.2s^2 + 2.85s + 0.90$$  \hfill (39)

Matching coefficients between the PI systems characteristic equation and required characteristic equation we get that $\tau = 0.3125, k_p = 0.8906, k_i = 0.2813$.

The first simulations run were used to find the best $k_p$ and $k_i$ values that provided the greatest amount of robustness without harshness and ensure stability for both the P and PI controller. The simulations were done assuming a typical passenger car in gear 5 with constants seen in the above table.
Figure 4: Simulation of a typical passenger car under varying $k_p$ (P Control)

As can be seen from figure 4 both $k_p = 0.75$ and $k_p = 1$ seem to be reasonable values without either being way too fast to react, producing harshness in the ride quality, or too slow as to not reach the desired velocity in a reasonable time. The $k_p = 0.75$ was chosen as it produced a slightly less harsh ride and still reached the desired velocity in about the same time as $k_p = 1$. 
Figure 5: Simulation of a typical passenger car under varying $k_p$ (PI control, $k_i = 0.2813$)
Similar to the method used to find $k_p$ for the P control system, varying $k_p$ and $k_i$ values were also simulated for the PI system (figure 5 and figure 6). Although other values are seen to have similar results, the values $k_p = 0.8906$ and $k_i = 0.2813$ were chosen for simplicity as they match the derived values, as well as provide the driver with the most effective acceleration and deceleration from 10 m/s to 20 m/s and down to 5 m/s.

Figure 6: Simulation of a typical passenger car under varying $k_i$ (PI control, $k_p = 0.8906$)
The next set of simulations were done to test the P and PI systems with varying gear ratios as associated with gear 4, 5, and 6 of a Hyundai Genesis Coupe ($m = 1600, r_{eff} = 0.483$). The simulations assumed the same changes in velocity as the previous simulations. The following gear ratios were used:

<table>
<thead>
<tr>
<th>Gear 4</th>
<th>Gear 5</th>
<th>Gear 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.143</td>
<td>0.867</td>
<td>0.691</td>
</tr>
</tbody>
</table>

Figure 7: Simulation of a Hyundai Genesis Coupe under varying gear ratio (P control)
As can be seen from Figure 7 and Figure 8, in order to keep the vehicle at the desired velocity no matter what gear the car is in the vehicle's engine increases the rotational speed of the engine and decreases the mass of the air in the engine as the car shifts up. It is also apparent that the PI system changes the rotational speed of the engine and the mass of air in the engine slightly faster than the simpler P system.

In the next set of simulations, the gear ratio was changed in the same manner as the above simulations assuming the same vehicle and gear ratios, but instead was simulated with velocities more closely related with driving on a highway in the United States. The vehicle was brought from 0 mph to 75 mph (33.5 m/s) down to 50 mph (22.4 m/s) and back up to 65 mph (29 m/s).
Figure 9: Simulation of a Hyundai Genesis Coupe under varying gear ratio on a United States highway (P control)

Figure 10: Simulation of a Hyundai Genesis Coupe under varying gear ratio on a United States highway (PI control)
The following simulations compare the response of a Suzuki motorcycle, Hyundai Genesis Coupe, and 18 Wheeler Semi-Tractor Trailer Truck on a typical United States highway. The simulations were done assuming that all the cars were in gear 5. The following gear ratios, effective tire radii, and masses were used for the different vehicles:

<table>
<thead>
<tr>
<th></th>
<th>Suzuki Motorcycle</th>
<th>Hyundai Genesis Coupe</th>
<th>18 Wheeler</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m ) (kg)</td>
<td>215</td>
<td>1600</td>
<td>36287</td>
</tr>
<tr>
<td>( R_{eff} ) (m)</td>
<td>0.406</td>
<td>0.483</td>
<td>0.572</td>
</tr>
<tr>
<td>( R )</td>
<td>1.3</td>
<td>0.867</td>
<td>3.42</td>
</tr>
</tbody>
</table>

![Figure 11: Simulation of a Suzuki Motorcycle, Hyundai Genesis Coupe, and 18 Wheeler Semi-Tractor Trailer Truck on a typical United States highway (P control)](image)
Figure 12: Simulation of a Suzuki Motorcycle, Hyundai Genesis Coupe, and 18 Wheeler Semi-Tractor Trailer Truck on a typical United States highway (PI control)

As can be seen from Figure 11 and Figure 12 the gear ratio is inversely proportional to the rotational speed of the engine. Thus, the Hyundai Genesis Coupe has the highest engine rotational speed and the 18 Wheeler Semi-Tractor Trailer Truck has the lowest.

The last set of simulations done compared the response of the P controller to the PI controller if a Hyundai Genesis Coupe were driving on the German Autobahn. The simulation assumes that the car is in gear 5 and that it starts from 0 mph, reaches the top speed of 155 mph (70 m/s), and then down to the Autobahns recommended speed of 80 mph (35 m/s).

Figure 13: Simulation of the comparison between P and PI control for a Hyundai Genesis Coupe on the German Autobahn
As can be seen from Figure 13 the PI controller has a faster response time than the simpler P controller.

### 4.2 Anti Lock Brakes

The following constants are used in the simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational Acceleration ( g )</td>
<td>( 32.2 \text{ft/s}^2 )</td>
</tr>
<tr>
<td>Maximum Brake Pressure ( P_b )</td>
<td>( 1500 \text{psi} )</td>
</tr>
<tr>
<td>Mass (1 wheel) ( m )</td>
<td>( 50 \text{ lbs} )</td>
</tr>
<tr>
<td>Radius of wheel ( R_r )</td>
<td>( 1.25 \text{ ft} )</td>
</tr>
<tr>
<td>Initial Velocity ( v_0 )</td>
<td>( 88 \text{ ft/s} )</td>
</tr>
<tr>
<td>Wheel rotational inertia ( I )</td>
<td>( 5 \text{ lbsin}^2 )</td>
</tr>
<tr>
<td>Piston area ( A_{\text{piston}} ), ( K )</td>
<td>( 1 \text{ ft}^3 )</td>
</tr>
<tr>
<td>Friction coefficient ( \mu )</td>
<td>( 0.1 - 0.85 )</td>
</tr>
</tbody>
</table>

The desired slip ratio is set to 0.2—a compromise between achieving a maximum friction coefficient during acceleration and braking, as well as turning [5].

The friction coefficient varies from 0.1 to 0.85 to simulate various road conditions (dry road, wet road, gravel, and ice).

For the second part of the simulation, a vehicle was run on the Autobahn, at an initial velocity of 155 mph.

For the controller, \( \eta = .4 \).
As the friction coefficient increases, it is evident that wheel and vehicle speeds decrease faster (here vehicle speed is equivalent to speed of the un-driven wheels, while wheel speed is taken as velocity of the driven wheel).
The next set of simulations were done with a step input friction coefficient. This is to simulate a vehicle on a dry road that suddenly hits a patch of ice (Figure 16(a)) or wet road (Figure 16(b)). The peaks in slip are clearly visible.

We can verify the validity of the controller by plotting the error over time. For this simulation, the error we want driven to zero is the difference between \( \lambda_{\text{act}} - \lambda_{\text{des}} \). For each friction coefficient, it is apparent that the error approaches zero after roughly seven seconds. The higher the friction coefficient, the easier it is for the controller to drive the error between slip ratios to zero (Figure 15).
The vehicle in this simulation experiences changing road conditions. Initially the vehicle is on a dry road (with $\mu = 0.85$) and after ten seconds the vehicle hits a patch of ice (a) or wet road (b). Again, the validity of the controller can be verified by noting the slip error drops to near zero after only 4 seconds initially, and then at ten seconds the controller must re-stabilize the vehicle and does so after a mere three seconds.

For this second set of simulations, $v_0 = 227.3\, ft/s$ (155 mph), to simulate a vehicle on the German Autobahn. We will examine how an ABS system handles high speeds.
Figure 18: Wheel speed, vehicle speed, and slip ratio vs time on the Autobahn [Blue marks control on; Red marks control off]

Compared with the vehicle traveling at $v_0 = 88 \text{ ft/s}$ (60 mph), this vehicle on the Autobahn sees a significantly longer time before stabilization of slip ratio. Also noticeable is the fact that the dynamic system without an ABS controller slips significantly faster at the higher speed. Even with the control turned on, the vehicle begins to slip out, and then is stabilized after roughly 15 seconds (a significant time increase from the three second response we saw at 88 ft/s.)
Figure 19: Wheel speed, vehicle speed, and slip ratio vs time for a stepped input friction coefficient on Autobahn (Blue marks control on; Red marks control off).

Figure 20: Error input for sliding mode controller, λ error vs. time, for vehicle travelling on Autobahn.
Figure 21: Error input for sliding mode controller, $\lambda$ error vs. time, for vehicle travelling on Autobahn, with stepped input $\mu$ (Blue marks $\mu = 0.1$; Red marks $\mu = 0.4$)

The vehicle in this simulation experiences changing road conditions. Initially the vehicle is on a dry road (with $\mu = 0.85$) and after ten seconds the vehicle hits a patch of ice (a) or wet road (b). The error goes to zero here again, although it does take longer to stabilize the system. The high speeds are presenting a physical challenge to the control system, because the wheels have a much higher potential to lock up due to the tendency of the wheels to lock up when a large break pressure is applied.

### 4.3 Automated Highway System

The following constants are used in the simulations unless noted otherwise.

\[
\begin{array}{c|c|c|c|c}
\tau & C_1 & \eta & w_n & L_i \\
2 & 0.9 & 2 & \frac{1}{10} & 9 \\
\end{array}
\]

The first simulations ran were ones to conclusively find a proper $K_p$ for the proportional controller of the lead car that provides the greatest amount of robustness and least amount of overshoot.
As can be seen, $K_p = 0.15$ provides the most reliable and constant acceleration without overshoot. It provides a slower ramp up to top speed, resulting in a more comfortable ride for the vehicle passenger.
A similar method was used to determine a suitable $K_i$ and $K_p$, as can be seen in Figure 23 and Figure 24. The values $K_p = 0.01$ and $K_i = 0.2$ provide the most comfortable acceleration of the lead car to the maximum cruising speed of 65 miles per hour (29 m/s).

In order to properly test the string stability of the platoon of vehicles under the lead vehicle’s two different controllers, they had to be simulated under a number of conditions. The robustness of the system was tested in multiple ways. First, the number of vehicles in the platoon was varied between 5 and 10 cars. Over the course of 15 minutes, the lead vehicle will accelerate from 0 miles per hour to 75, down to 40, up to 65, and then down to 0. These conditions were found to best simulate varying speed limits on a United States highway. Each vehicle is placed 100 meters away from each other for the starting position in this simulation, with the lead car being at 500 meters and the fifth car located at 0 meters.
As is most apparent in Figure 25(b), after about 2 minutes the platoon becomes string stable, and stays so for the rest of the simulation under Proportional Control.

Utilizing the same control scheme, but with 10 vehicles in the platoon instead of 5, with the lead car being at 1000 meters and the tenth car at 0 meters. A pattern begins to develop:
Figure 26: Ten Car Platoon Simulations (Proportional Control)

Figure 26(b) shows that, as more vehicles are added to the platoon, the speed of the last car severely overshoots the speed of the lead car. However, Figures 26(a) and 26(c) show that this change in starting velocity has very little effect on the position and acceleration of this final car.

Using the PI control derived earlier, we get similar results.
The only difference apparent here is that the last car has more a slightly varied velocity that takes more time to settle after each change in cruising speed. This is especially apparent when accelerating down to the final velocity of zero meters per second.
Figure 28: Ten Car Platoon Simulations (PI Control)

Similar to the Proportional control with 10 cars, the PI controller also allows for a lot of overshoot by the final car. However, in this case, Car 10 reaches almost 50 meters per second after the initial acceleration. Also, it has a sharper negative acceleration when approaching the final speed of zero meters per second.

Another major factor that can be modified is the spacing constant $L_i$ within the sliding surface. For the previous simulations, $L_i$ has been set to a default value of 9. However, by varying this, we can achieve different results. The simulations (for a proportional controller with $K_p = 0.15$) are noted in Figure 29(a).
By utilizing a greater $L_i$ value, the velocities of the vehicles begin to match up perfectly at 75 mph (33 m/s) after the initial acceleration, resulting in no overshoot by the final car in the platoon. Combining the most optimal value of $L_i = 100$ with the ten car platoon with proportional control, we get the following results.
Figure 30: Ten Car Platoon Simulations with $L_i = 100$ (Proportional Control)

Figure 30(b) no longer possess the overshoot apparent in Figure 26(b) because of the modified $L_i$ value. Logically, as seen in Figure 30(a), each car covers less distance than the lead car, but this is simply because of $L_i$ changing the spacing maintained by the sliding control throughout the simulation. With this optimization in place to allow for greater spacing we can further test the maximum speeds that the platoon can reach and still be stable.

In a practical example, a platoon of cars could be utilized on the German Autobahn to effectively transition from regulated to unregulated speed limits, such as between construction zones or areas limited for reduced pollution and noise. As such, the following simulation showcases a ten car platoon traveling from 0 mph up to 155 mph (70 m/s), the electronically limited max speed of most vehicles, down to a construction
area speed limit of 50 mph (22 m/s), back up to the Autobahn’s ”recommended” speed of 80 mph (35 m/s),
before finally settling back at 155 mph.

Figure 31: Ten Car Platoon Autobahn Simulations (Proportional Control)
Similar results are achieved by the proportional and PI controller. Under the PI controller, the platoon slightly overshoots the lead car’s example (Figure 32(b)), taking slightly longer to accelerate to it’s cruising speed. However, under the proportional control, the platoon stays relatively true to the lead car (Figure 31(b)), following its velocity almost exactly.
5 Analysis

5.1 Cruise Control

Cruise control is a fairly common example of longitudinal control available on most cars today. The driver sets the desired speed and the cruise control system automatically controls the throttle to maintain the desired speed. It is however, the drivers responsibility to ensure that the vehicle can safely travel at the speed to which it is set. It is important that a cruise control system have a fast and accurate response, as well as, an easy and effective way to remove the control the system has on the throttle of the car. The robustness of the system is determined by the type of controller used in the upper lay controller.

The P, or proportional, controller makes a change to the output that is proportional to the current error value. It is adjusted by multiplying the error by a constant \( k_p \), which improves the rise time of the system. As can be seen from the simulations above, if the proportional gain is high, there is large change in output for the given change in error. In some cases the system can become unstable. In contrast, a small \( k_p \) value results in a small change in output for a given change in input error. Thus, the system becomes less responsive. It is possible for the system control action to be too small when responding to disturbances if the gain is too low. The proportional controller may not settle to its target value.

In order to eliminate the residual steady state error that results from a pure P controller, an I, or integral, controller can be added to the P controller producing a PI controller. The integral controller is the sum of the instantaneous error over time, thus giving the offset that should have been corrected previously. The sum of the error is multiplied by the integral gain \( k_i \) and added to the controller output. Since the integral term responds to accumulated error, it is possible for the term to present an overshoot as the system attempts to settle to the users desired input.

Cruise control is seen to have both advantages and disadvantages. It is advantageous in that it is useful for long drives, thus reducing the drowsiness of the driver and allowing more comfort for the driver as it is easier to move around in their seat. It results in higher fuel efficiency as the amount of throttle used by the car is more stable than if the driver were to be in control for a long period of time. Cruise control is also used by drivers to avoid speed limit violations. Many times a driver will increase their speed without noticing if they have been driving on a highway for an extended period of time. The cruise control system is, however, not perfect and does not have an instantaneous reaction time as the speed of the car fluctuates when going up or down hills. The speed to which the system reacts to such sudden changes depends on the controller used (P or PI).
One disadvantage of cruise control is that the lack of need to control the pedal pressure can cause the driver to forget that he is still driving after a long period of time and may lead to accidents if there are sudden obstacles or perturbances such as a vehicle entering the highway into the lane that the drivers vehicle is currently in. Another disadvantage is that when used weather conditions such as rain or snow where the coefficient of friction of the road is decreased, the vehicle if kept at the typical constant speed for a dry day has the chance of going into a skid. Stepping on the brake rashly so as to disengage the cruise control system and regain control of vehicle may lead to the driver having even less control of the vehicle as seen in the analysis of an ABS system below. A final disadvantage to cruise control is that if the vehicle is driving over hilly terrain the cruise control system tends to over throttle the engine while going uphill and retard the throttle while going downhill due to the fact that it is constantly adjusting the throttle due the so many sudden perturbances in the system. The system doesnt have enough time or barely has enough time to stabilize the speed of the vehicle before another perturbances is introduced. This strains the engine and may cause premature wear.

In order to determine the situations in which cruise control will be beneficial and the situations in which it will not be, it is necessary to run simulations similar to the ones above. It is important to determine the control gains $k_p$ and $k_i$ such that the system is stable, robust, and produces a suitable ride quality to the passengers. Both the P and PI systems can be stable, but as seen from simulations the PI system is more robust. It reacts to perturbances in the system faster than the simple P system while still producing about the same ride quality.

5.2 Anti Lock Brakes

ABS controls the vehicle by preventing the wheels from locking up during periods of hard braking, by maximizing the braking forces applied by each tire. This is done by maximizing the longitudinal force when possible, and preventing the applied force from surpassing this value. Physically, the ABS system limits the amount of brake torque applied to the wheels, in order to prevent over-braking, during which the wheels lock up and the braking force decreases (Figure 33). In the third quadrant, one can see the braking force hits a maximum, and then begins to decrease. The goal of ABS is to keep the braking force within a close range to the maximum, to prevent the wheels from locking and brake the vehicle reasonable safely. Braking too fast would result in a significantly harsh jerk, which is expectedly undesirable in a family vehicle. Therefore the goal of ABS is to produce smooth, effective braking.
By comparing wheel speed to vehicle speed (both of which can be measured), the imminent locking of the tires can be detected (by detecting a sharp increase in slip ratio) and the ABS control system will turn on and take control of the brakes to decrease the braking force and drop it back to the maximum force range, in order to provide more tire force and brake successfully.

Figure 33: Tire force as a function of tire slip. ABS control tries to maximize tire force by controlling tire slip during hard braking [2].

For each simulation, the velocity of the uncontrolled vehicle decreases much more rapidly than the controlled vehicle, but with good reason. Once the wheels lock, they experience an increase in frictional force, and thus experience deceleration due to the skidding of the wheels on the road. With the control on, the vehicle decreases much slower, and much safer. Because the goal of ABS is to increase safety in the way of controlling brake force, it is desirable to have a smaller deceleration rate, which will assuredly prevent the wheels from locking and will produce a smoother, safer chassis ride.

For constant road conditions (Figure 14), with ABS control the error stabilizes rather quickly (within the first 5 seconds) at zero. Without control, the slip ratio jumps to unity within the first three seconds, wheel speed drops to zero, and the vehicle begins to skid, decreasing its velocity significantly. With each simulation, the friction coefficient was increased to simulate different road conditions. At $\mu = 0.1, 0.4, 0.6, 0.85$, the road
conditions are: ice, wet road, gravel, and dry road respectively.

For non-constant road conditions (Figure 16) the vehicle with ABS control decreased consistently at a steady rate. The changing friction coefficient at t=10 seconds did not disrupt the vehicles deceleration noticeably. However, without the controller on, the vehicle presumably does notice the slicker road, as can be seen in the slip subplots of Figure 16. The slip ratio jumps after hitting the second road surface, which would be noticeable to the one driving the car. This is relatively undesirable seeing as humans have an unpredictable nature when it comes to handling surprise situations. The driver could be surprised by the wheels beginning to slip on the ice or wet road, and swerve, hitting a pedestrian, another car, or a passive object (like a divider or street sign). Vehicle control systems must be centered around the user, because of the active input of a human driver, and thus continuity of the ABS system is important when it experiences different road conditions.

For the simulations with the car on the Autobahn, the initial velocity was significantly higher than during the initial simulations (155 mph vs. 60 mph) and the differences are observable. The controller needs to actuate the brakes at a much slower rate in order to prevent the brakes from locking, and thus the difference in initial and final velocities (ΔV) on the Autobahn is smaller than at the more reasonable initial velocity of 60 mph. It is also apparent that once the controller stabilizes the slip ratio at λ_{des}, the velocity of the wheels actually increases slightly. This can be explained by the fact that as the controller detects slip and begins to actuate the brake pressure it alleviates a little bit of brake pressure in order to bring P_{br} back to within the maximum range. This would slightly change the wheel speed, because the hard braking would slow down the wheels, which can be seen in the plots, and then when the brake pressure is stabilized at a maximum, the wheel speed increases slightly.

The ABS controller was able to completely modulate the slip ratio, even when implemented on a vehicle at high speed and on an icy road. There was a relatively significant time delay between brake initiation and stabilization (in the range of 15 seconds) for the most complicated situation presented, but nevertheless the controller was able to stabilize the slip ratio and prevent the wheels from locking and skidding (Figure 18).

It is important to note that this simulation is not designed to simulate real-world conditions; it has been heavily simplified to increase ease of use, although currently its validity on a real road surface can be disputed. In the control law derivation, all resistances to motion are ignored except friction, for simplification. Model parameters were chosen as typically standard values, although many real-life conditions were omitted from this simulation. Actual experimental testing should be conducted to verify the validity, stability and robustness of ABS systems, although these simulations do show overall success of the model.
A continuation of this study would include using possible implementation of PID control to compare with sliding mode, to see which controller could drive the errors to 0 faster. This is desirable to maximize braking force as early as possible, as well as to keep the slip ratio as close to the desired slip ratio as possible. Further analysis could come from looking at ABS on different types of vehicles and adjusting control gain parameters to effectively maximize the efficiency of the ABS system.

5.3 Automated Highway System

An automated highway system could provide a long lasting impact on all industrial countries that utilize some form of highway or motorway. However, the main concern with creating such a highway system is safety. What if the cars crash? What if a system fails? For this very reason, thousands of simulations must be done to find a suitable control system that will work under a variety of conditions without fail, while remaining practical and effective. The first such tests helped to decide what controller system would be most reliable for adaptive cruise control within an automated highway system. After testing various gain control constants, a $K_p$ and $K_i$ was found that provided gradual acceleration and least amount of overshoot. These two characteristics combine to provide safety and ride quality to the passenger, ensuring that they are not unnecessarily jerked around.

5.3.1 Benefits

There are many benefits to utilizing an automated highway system, the biggest being increased roadway capacity. By increasing the number of vehicles per hour per lane (by means of standardizing speeds through specific sections of highway), far more vehicles could fit on specific parts of a motorway. Inefficiencies would be eliminated, such as random lane changes and inopportune, that cause congestion on highways today. However, the largest contribution to traffic are accidents, mostly caused by human error, which would be completely removed in an automated highway system. Other benefits also include more efficient energy consumption as a result of going at constant speeds (as well as improved air quality), as well as the possibility of more compact roadways, as less lanes would be necessary. To the end consumer, an automated highway system would provide a safe, effective way to get to wherever one needs to go without congestion, allowing one to get to work quickly and easily. Also, if a car were to stall, the sliding control would cause all following cars to stop as a safety precaution.

Ride quality is another one of the benefits of utilizing an automated highway control system. By analyzing
both the P and PI control systems and making modifications to the necessary constants \((k_p \text{ and } k_i)\), the most comfortable and accurate control system can be implemented. By simulating these constants in Section 4.3, the control system of the lead car is able to comfortably reach a top speed of 65 mph without alarming the passenger or causing unnecessary discomfort. Also, as opposed to other tested values, the values settled upon do not cause overshoot, possibly breaking speed limits depending on where the vehicle is driven.

5.3.2 Impact

Despite the benefits described, there can be serious problems with an automated highway system if not properly addressed or accounted for. When merging onto and off of a motorway in a non automated highway system, drivers blindly enter the highway, expecting other drivers to make room in the merging lane. If the section of motorway is already slightly congested, the braking by other motorists to let mergers in could exacerbate the congestion, entering into a vicious cycle. With an automated highway system there would be the problem of transitioning from an automated section to non-automated roads, possibly causing a bottleneck, removing any potential timesaving benefits that the automated sections provided. However, one possible way to offset the entry bottleneck is to increase the spacing maintained by the sliding control. This allows for easier entry into a platoon, as well as enhanced string stability as the size and speed of the platoon grows, as seen in Section 4.3. [3]

Safety is a major concern as well. If the platoon is not string stable, terrible chain reactions could occur. If an error is allowed to propagate over the course of an entire platoon, the vehicles could easily catch up to one another, causing congestion or even collisions. In order to properly test for string stability, the platoon was run under a number of different tests, including addition of vehicles, high speeds, and sudden change in speeds, as well as a practical application (the German Autobahn). Over the course of each test, once the spacing constant \(L_i\) was modified, each vehicle in the platoon properly followed the lead vehicle’s direction, maintaining proper string stability and robustness for each of the simulated conditions. [6]

However, such a large spacing constant as chosen in these simulations could prove hazardous as well. With such a large space in between cars maintained by the sliding control (with \(L_i = 100\)), it could take longer for cars near the end of the platoon to reach a destination if there is a sudden influx of cars merging in a downtown city zone. Each new vehicle that would merge would cause cars already in the platoon to have to slow down to generate the spacing between each new car and the already established vehicles, once again possibly offsetting all time saving possibilities.
6 Conclusion

6.1 Cruise Control

By simulating a P and PI controller with varying $k_p$ and $k_i$ gains for a cruise control system, the gain values which produced stability and robustness to the system while still ensuring a good ride quality were chosen. Both systems were tested for different variations in velocity, varying gear ratio, and varying type of vehicle the system was implemented on. The simulations were done using an upper and lower layer controller. The upper layer implemented either a P or PI controller and the lower layer implemented a sliding mode control. It was seen that the PI controller was more robust than the P controller. The PI controller improves rise time and reduces the steady state error. Cruise control systems are convenient for long drives, but place a strain on the engine when the vehicle is on hilly terrain. Due to their ability to increase fuel efficiency and provide comfort to drivers, cruise control systems are placed on most vehicles today.

6.2 Anti Lock Brakes

Through multiple simulations, the robustness and stability of this ABS control system is verified. With control on, the vehicle did not slip out when on slick roads, or at high speeds, or with these two parameters combined and applied to the vehicle. The ABS control managed to drive the error to zero, while preventing the wheels from slipping. When the control is turned off, the wheels lock within the first few seconds, and the vehicle presumably screeches to a halt after a score of seconds. To improve ride quality, safety, and driver experience, ABS controllers are now being implemented on most road vehicles, for good reason.

6.3 Automated Highway System

Over the course of multiple simulations, proper controller gains for a PI and proportional controller were found for a lead car of an automated highway system. The proportional controller provided a more stable ride quality, and was therefore implemented into a platoon. The platoon implemented a basic sliding control, and was tested for a number of conditions, including adding vehicles, changing top speeds, and changing spacing. Modifications were made in order to remove any overshoot or ride quality concerns, as well as to ensure that the lead vehicle and platoon would stay string stable over the course of a simulation. With the adjustments made, the system could easily be implemented once societal and environment concerns are addressed.
References


